

Centre Number						Candidate Number				
Surname										
Other Names	MR BARTON'S									
Candidate Signature	WORKED SOLUTIONS.									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	



General Certificate of Education  
Advanced Level Examination  
June 2015

# Mathematics

# MS2B

## Unit Statistics 2B

Friday 12 June 2015 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Time allowed

- 1 hour 30 minutes

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J U N 1 5 M S 2 B 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

- 1** In a survey of the tideline along a beach, plastic bottles were found at a constant average rate of 280 per kilometre, and drinks cans were found at a constant average rate of 220 per kilometre. It may be assumed that these objects were distributed randomly and independently.
- Calculate the probability that:
- (a) a 10 m length of tideline along this beach contains no more than 5 plastic bottles; [2 marks]
  - (b) a 20 m length of tideline along this beach contains exactly 2 drinks cans; [3 marks]
  - (c) a 30 m length of tideline along this beach contains a **total** of at least 12 but fewer than 18 of these two types of object. [4 marks]

QUESTION  
PART  
REFERENCE

Answer space for question 1

a)  $280 \text{ per km} = 2.8 \text{ per } 10 \text{ m}$   
 $\rightarrow PB \sim Po(2.8)$   
 $P(PB \leq 5) = 0.9349$  (from table)

b)  $220 \text{ per km} = 4.4 \text{ per } 20 \text{ m}$   
 $\rightarrow DC \sim Po(4.4)$   
 $P(DC = 2) = e^{-4.4} \times \left(\frac{4.4^2}{2!}\right)$   
 $= 0.118844\dots$

c)  $Total = 500 \text{ per km} = 15 \text{ per } 30 \text{ m}$   
 $\rightarrow T \sim Po(15)$   
 $P(12 \leq T < 18)$   
 Need 12, 13, 14, 15, 16, 17  
 $\rightarrow P(T \leq 17) - P(T \leq 11)$



QUESTION  
PART  
REFERENCE

Answer space for question 1

$$= 0.749 - 0.1848$$
$$= 0.5641$$

Turn over ►





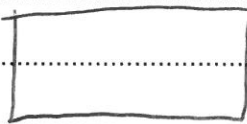
- 2 The continuous random variable  $X$  has probability density function defined by

$$f(x) = \begin{cases} \frac{1}{k} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

- (a) Write down, in terms of  $a$  and  $b$ , the value of  $k$ . [1 mark]
- (b) (i) Given that  $E(X) = 1$  and  $\text{Var}(X) = 3$ , find the values of  $a$  and  $b$ . [4 marks]
- (ii) Four independent values of  $X$  are taken. Find the probability that exactly one of these four values is negative. [3 marks]

QUESTION  
PART  
REFERENCE

Answer space for question 2

a) Area = 1  $\frac{1}{k}$    
 $b - a$

$$\rightarrow k = (b - a)$$

b) 1) ①  $E(X) = \frac{1}{2}(a + b) = 1$

②  $\text{Var}(X) = \frac{1}{12}(b - a)^2 = 3$

②  $\rightarrow (b - a)^2 = 36$

$b - a = 6$  (cannot be  $-6$  as  $b > a$ )

①  $\rightarrow a + b = 2$

$b - a = 6$

+  $b + a = 2$

$2b = 8 \rightarrow b = 4$

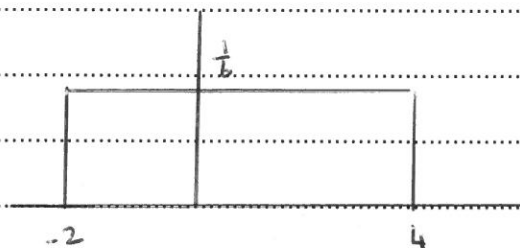
$a = -2$



QUESTION  
 PART  
 REFERENCE

## Answer space for question 2

ii)



$$P(X < 0) = 2 \times \frac{1}{6} = \frac{1}{3}$$

 use binomial, with  $P(\text{success}) = \frac{1}{3}$ 

$$\begin{aligned} \rightarrow P(S=1) &= {}^4C_1 \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^3 \\ &= \frac{32}{81} \text{ or } 0.39506 \end{aligned}$$

Turn over ►



- 3 A machine fills bags with frozen peas. Measurements taken over several weeks have shown that the standard deviation of the weights of the filled bags of peas has been 2.2 grams.

Following maintenance on the machine, a quality control inspector selected 8 bags of peas. The weights, in grams, of the bags were

910.4 908.7 907.2 913.2 905.6 911.1 909.5 907.9

It may be assumed that the bags constitute a random sample from a normal distribution.

- (a) Giving the limits to **four** significant figures, calculate a 95% confidence interval for the mean weight of a bag of frozen peas filled by the machine following the maintenance:
- (i) assuming that the standard deviation of the weights of the bags of peas is known to be 2.2 grams; [4 marks]
- (ii) assuming that the standard deviation of the weights of the bags of peas may no longer be 2.2 grams. [4 marks]
- (b) The weight printed on the bags of peas is 907 grams. One of the inspector's concerns is that bags should not be underweight.

Make **two** comments about this concern with regard to the data and your calculated confidence intervals.

[2 marks]

QUESTION  
PART  
REFERENCE

Answer space for question 3

a) i) From calc:  $\bar{x} = 909.2$

From question:  $\sigma = 2.2$ ,  $n = 8$

We know  $\sigma$ , so use Z

95% CI multiplier =  $\times 1.96$

$\rightarrow 909.2 \pm 1.96 \times \frac{2.2}{\sqrt{8}}$

$= (907.7, 910.7)$



QUESTION  
 PART  
 REFERENCE

## Answer space for question 3

ii) We no longer know  $\sigma$ , so need "s"  
 from calculator  
 $\rightarrow s = 2.39165$

As we don't know  $\sigma$  and  $n = 8$ , we need  
 to use  $t$

$$v = 8 - 1 = 7$$

$t_7$  95% multiplier = 2.365

$$\rightarrow 909.2 \pm 2.365 \times \frac{2.39165}{\sqrt{8}}$$

$$= (907.2, 911.2)$$

b)  Both confidence intervals are above 907,  
 so the claim about the AVERAGE weight is  
 probably valid

one of the weights in the sample (905.6) is  
 underweight, so no guarantee all weights  
 are greater than 907

Turn over ►





- 4 Wellgrove village has a main road running through it that has a 40 mph speed limit. The villagers were concerned that many vehicles travelled too fast through the village, and so they set up a device for measuring the speed of vehicles on this main road. This device indicated that the mean speed of vehicles travelling through Wellgrove was 44.1 mph.  $H_0$

In an attempt to reduce the mean speed of vehicles travelling through Wellgrove, life-size photographs of a police officer were erected next to the road on the approaches to the village. The speed,  $X$  mph, of a sample of 100 vehicles was then measured and the following data obtained.

$$\sum x = 4327.0 \quad \sum (x - \bar{x})^2 = 925.71$$

- (a) State an assumption that must be made about the sample in order to carry out a hypothesis test to investigate whether the desired reduction in mean speed had occurred. [1 mark]
- (b) Given that the assumption that you stated in part (a) is valid, carry out such a test, using the 1% level of significance. [8 marks]
- (c) Explain, in the context of this question, the meaning of:
- (i) a Type I error;
- (ii) a Type II error. [2 marks]

QUESTION  
PART  
REFERENCE

Answer space for question 4

a) The sample of vehicles chosen must be RANDOM

b)  $H_0: \mu = 44.1$   
 $H_1: \mu < 44.1$  (ITT)

$$\bar{x} = \frac{4327}{100} = 43.27$$

$$s^2 = \frac{925.71}{99} = 9.3506... =$$

$$\rightarrow s = 3.0578...$$





QUESTION  
PART  
REFERENCE

Answer space for question 4

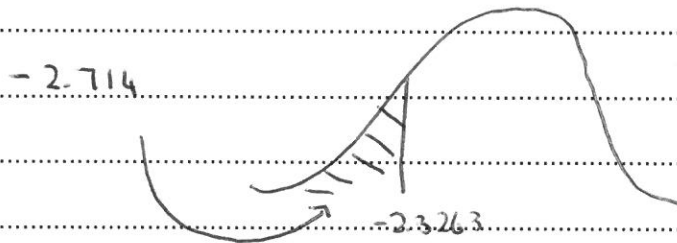
$n > 30 \rightarrow$  use  $Z$  because of  
Central Limit Theorem

**TEST STATISTIC**

$$Z = \frac{43.27 - 44.1}{3.0578 / \sqrt{100}} = -2.714$$

**CRITICAL VALUE**

1%, 1 tail ~~two~~ test,  $Z \rightarrow -2.3263$



$$-2.714 < -2.3263$$

$\rightarrow$  Reject  $H_0$

There is evidence at 1% significant level  
that the mean speed has reduced

c) i) **TYPE 1** Concluding the mean speed has  
reduced when it has not (Reject  $H_0$ )

**TYPE 2** Accepting mean speed is still  
44.1 kmph when it has in fact reduced  
(accept  $H_0$ )

Turn over  $\blacktriangleright$



- 5 In a particular town, a survey was conducted on a sample of 200 residents aged 41 years to 50 years. The survey questioned these residents to discover the age at which they had left full-time education and the greatest rate of income tax that they were paying at the time of the survey.

The summarised data obtained from the survey are shown in the table.

Greatest rate of income tax paid	Age when leaving education (years)			Total
	16 or less	17 or 18	19 or more	
Zero	32	3	4	39
Basic	102	12	17	131
Higher	17	5	8	30
<b>Total</b>	151	20	29	200

- (a) Use a  $\chi^2$ -test, at the 5% level of significance, to investigate whether there is an association between age when leaving education and greatest rate of income tax paid. [9 marks]
- (b) It is believed that residents of this town who had left education at a later age were more likely to be paying the higher rate of income tax. Comment on this belief. [1 mark]

QUESTION  
PART  
REFERENCE

Answer space for question 5

a)  $H_0$ : No association between age at which they left education and the rate of income tax they pay. (Independent)  
 $H_1$ : There is an association. (Non-Independent)

Expected

	<16	17 or 18	19 or more
Z	29.645	(3.9)	5.655
B	98.905	13.1	18.995
H	22.65	(3)	(4.35)



QUESTION  
PART  
REFERENCE

Answer space for question 5

3 values  $< 5 \rightarrow$  combine '17 or 18' and  $\geq 19$  into " $\geq 17$ "

New Expected

	$< 16$	$\geq 17$
Z	29.445	9.555
B	98.905	32.095
H	22.65	7.35

CHI-SQUARED

	$< 16$	$\geq 17$
$\frac{\sum(O - E)^2}{E}$	0.221	0.6872
B	0.0969	0.2984
H	1.4093	4.3431

$\rightarrow$  Test Statistic = 7.05

CRITICAL VALUE

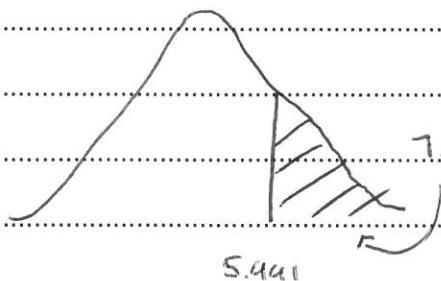
$$v = (3-1)(2-1) = 2$$

5%

$\rightarrow 5.991$

$$7.05 > 5.991$$

$\rightarrow$  Reject  $H_0$



7.05 Evidence at 5% level that there IS AN ASSOCIATION between age of leaving education & tax paid

Turn over ►





QUESTION  
PART  
REFERENCE

## Answer space for question 5

b) Need to refer to data:  
From Observed: 8 students aged 19 or  
more paid higher tax  
From Expected: that number was 4.35  
∴ The belief is supported



- 6 The continuous random variable  $X$  has the cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}x - \frac{1}{16}x^2 & 0 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

- (a) Find the probability that  $X$  lies between 0.4 and 0.8 .

[2 marks]

- (b) Show that the probability density function,  $f(x)$ , of  $X$  is given by

$$f(x) = \begin{cases} \frac{1}{2} - \frac{1}{8}x & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

[1 mark]

- (c) (i) Find the value of  $E(X)$  .

[3 marks]

- (ii) Show that  $\text{Var}(X) = \frac{8}{9}$ .

[4 marks]

- (d) The continuous random variable  $Y$  is defined by

$$Y = 3X - 2$$

Find the values of  $E(Y)$  and  $\text{Var}(Y)$  .

[2 marks]

 QUESTION  
PART  
REFERENCE

## Answer space for question 6

(6) a)  $P(0.4 < X < 0.8)$   
 $= F(0.8) - F(0.4)$   
 $= \left[ \frac{0.8}{2} - \frac{0.8^2}{16} \right] - \left[ \frac{0.4}{2} - \frac{0.4^2}{16} \right]$   
 $= 0.36 - 0.19 = 0.17$

b)  $\frac{dF(x)}{dx} = f(x) \rightarrow \frac{1}{2} - \frac{2}{16}x$   
 $= \frac{1}{2} - \frac{1}{8}x$



QUESTION  
 PART  
 REFERENCE

## Answer space for question 6

$$a) \ i) \ E(x) = \int_0^4 x f(x) dx$$

$$= \int_0^4 \left( \frac{1}{2}x - \frac{1}{8}x^2 \right) dx$$

$$= \left[ \frac{1}{4}x^2 - \frac{1}{24}x^3 \right]_0^4$$

$$= \frac{1}{4}(4^2) - \frac{1}{24}(4^3) - 0$$

$$= 4 - \frac{8}{3} = \frac{4}{3}$$

$$ii) \ \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\Rightarrow E(X^2) = \int_0^4 x^2 f(x) dx$$

$$= \int_0^4 \left( \frac{1}{2}x^2 - \frac{1}{8}x^3 \right) dx$$

$$= \left[ \frac{1}{6}x^3 - \frac{1}{32}x^4 \right]_0^4$$

$$= \frac{1}{6}(4^3) - \frac{1}{32}(4^4) - 0$$

$$= \frac{32}{3} - 8 = \frac{8}{3}$$

$$\rightarrow \text{Var}(X) = \frac{8}{3} - \left[ \frac{4}{3} \right]^2 = \left( \frac{8}{9} \right)$$





QUESTION  
PART  
REFERENCE

Answer space for question 6

$$d) Y = 3X - 2$$

$$\begin{aligned} E(Y) &= 3E(X) - 2 \\ &= 3\left(\frac{4}{3}\right) - 2 = 2 \end{aligned}$$

$$\begin{aligned} \text{Var}(Y) &= 3^2 \text{Var}(X) \\ &= 9\left(\frac{8}{9}\right) = 8 \end{aligned}$$



- 7 Each week, a newsagent stocks 5 copies of the magazine *Statistics Weekly*. A regular customer always buys **one** copy. The demand for **additional** copies may be modelled by a Poisson distribution with mean 2.

The number of copies sold in a week,  $X$ , has the probability distribution shown in the table, where probabilities are stated correct to three decimal places.

$x$	1	2	3	4	5
$P(X=x)$	0.135	0.271	0.271	$a$	$b$

- (a) Show that, correct to three decimal places, the values of  $a$  and  $b$  are 0.180 and 0.143 respectively. [3 marks]
- (b) Find the values of  $E(X)$  and  $E(X^2)$ , showing the calculations needed to obtain these values, and hence calculate the standard deviation of  $X$ . [5 marks]
- (c) The newsagent makes a profit of £1 on each copy of *Statistics Weekly* that is sold and loses 50 p on each copy that is not sold. Find the mean weekly profit for the newsagent from sales of this magazine. [2 marks]
- (d) Assuming that the weekly demand remains the same, show that the mean weekly profit from sales of *Statistics Weekly* will be greater if the newsagent stocks only 4 copies. [5 marks]

 QUESTION  
PART  
REFERENCE

## Answer space for question 7

⑦ a)  $A \sim Po(2)$   $A = \text{Additional copies}$

So, for  $x = 4$ , we need  $A = 3$

$$P(A=3) = \frac{e^{-2} \times 2^3}{3!} = 0.1804\dots$$

$$= 0.180 \text{ (3dp)}$$

~~For  $x = 5$ , we need  $A = 4$~~

~~$$P(A=4) = \frac{e^{-2} \times 2^4}{4!}$$~~



QUESTION  
PART  
REFERENCE

Answer space for question 7

$$\text{For } x = 5, \text{ need } 1 - (0.135 + 0.271 + 0.271 + 0.1804) \\ = 0.143$$

$$\text{b) } E(x) = (1 \times 0.135) + (2 \times 0.271) + (3 \times 0.271) \\ + (4 \times 0.180) + (5 \times 0.143) \\ = 2.925$$

$$E(x^2) = (1^2 \times 0.135) + (2^2 \times 0.271) + (3^2 \times 0.271) \\ + (4^2 \times 0.180) + (5^2 \times 0.143) \\ = 10.113$$

$$\text{VAR}(x) = 10.113 - 2.925^2$$

$$\Rightarrow \text{SD}(x) = \sqrt{10.113 - 2.925^2} \\ = 1.25$$

c)	x	1	2	3	4	5
	PROFIT	-1	0.5	2	3.5	5
	PROB	0.135	0.271	0.271	0.180	0.143

$$\text{Mean profit} = E(\text{profit}) = (-1 \times 0.135) + (0.5 \times 0.271) \\ + (2 \times 0.271) + (3.5 \times 0.180) \\ + (5 \times 0.143) = \pounds 1.89$$

Turn over ►





QUESTION  
 PART  
 REFERENCE

## Answer space for question 7

d) New distribution:

$x$	1	2	3	4
Profit	-0.5	1	2.5	4
Prob	0.135	0.271	0.271	0.323

$$\begin{aligned}
 E(\text{Profit}) &= (-0.5 \times 0.135) + (1 \times 0.271) \\
 &\quad + (2.5 \times 0.271) + (4 \times 0.323) \\
 &= \pounds 2.17
 \end{aligned}$$

$\pounds 2.17 > \pounds 1.89$ , so profit is greater.

